

Field and phenomenological description of absorption of particles

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Abstract

Complicated processes in the medium (decays, reactions and $n\bar{n}$ conversion) involving final state absorption are considered. The calculations in the framework of field and phenomenological approaches are compared. The reasons for disagreement are studied. The field approach can tend to increase of total process probability as well as probability of channel corresponding to absorption, in comparison with phenomenological model.

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The multistep processes in the medium are frequently attended by the absorption of particles in the intermediate or final states. The simplest two-step process of this type is the $n\bar{n}$ conversion [1,2] in nuclear matter:

$$n \rightarrow \bar{n} \rightarrow M, \quad (1)$$

here M are the annihilation mesons. The qualitative process picture is as follows. The free-space $n\bar{n}$ conversion comes from the exchange of Higgs bosons with a mass $m_H > 10^5 \text{ GeV}$ [1,2]. From the dynamical point of view this is a momentary process: $\tau \sim 1/m_H < 10^{-29} \text{ s}$. The antineutron annihilates in a time $\tau \sim 1/\Gamma \sim 10^{-24} \text{ s}$, where Γ is the annihilation width of \bar{n} ; $\Gamma \sim 100 \text{ MeV}$.

Phenomenologically, the absorption of particles in the medium is described by ImU_{opt} , where U_{opt} is the optical potential. First of all we consider the antineutron annihilation by means of field approach. Let U_n and $U_{\bar{n}}$ are the real potentials of n and \bar{n} , respectively. The background nuclear matter field U_n is included in the neutron wave function: $n(x) = V^{-1/2} \exp(-i(\mathbf{p}_n^2/2m + U_n)t + i\mathbf{p}_n \mathbf{x})$. The interaction Hamiltonian is

$$\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + V\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}} + \mathcal{H}_a, \quad (2)$$

$$\mathcal{H}_{n\bar{n}} = \epsilon\bar{\Psi}_{\bar{n}}\Psi_n + H.c., \quad (3)$$

$V = U_{\bar{n}} - U_n$. Here $\mathcal{H}_{n\bar{n}}$ and \mathcal{H}_a are the Hamiltonians of $n\bar{n}$ conversion and annihilation, respectively; ϵ is a small parameter.

The corresponding diagram is shown in Fig. 1. The antineutron Green's function has the form

$$G = \frac{1}{G_0^{-1} - V} = -\frac{1}{V}, \quad (4)$$

$G_0^{-1} = \epsilon_n - (\mathbf{p}_n^2/2m + U_n) = 0$. The process amplitude is

$$M = -\epsilon G M_a = \epsilon \frac{1}{V} M_a. \quad (5)$$

Here M_a is the annihilation amplitude. For the process width Γ_f one obtains

$$\Gamma_f = N \int d\Phi |M|^2 = \frac{\epsilon^2}{V^2} N \int d\Phi |M_a|^2 = \frac{\epsilon^2}{V^2} \Gamma. \quad (6)$$

The normalizing multiplier N is the same for Γ_f and Γ .

In the phenomenological approach the \bar{n} -medium interaction is described by optical potential U_{opt} . Instead of Hamiltonian (2) we have

$$\mathcal{H}_I = \mathcal{H}_{n\bar{n}} + V\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}} - i\Gamma/2\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}}, \quad (7)$$

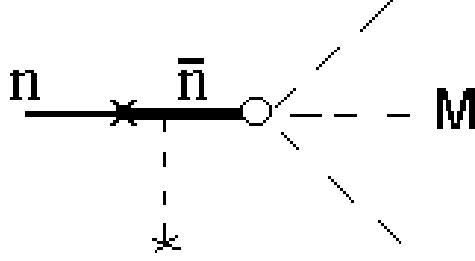


Figure 1: $n\bar{n}$ transition in nuclear matter. The antineutron annihilation is illustrated by circle.

$ImU_{opt} = -\Gamma/2$, $ReU_{opt} = V$. As a rule, this model is realized by means of equation of motion [3-5]. To avoid questions connected with the formalism, we use the diagram technique, as for the model (2). Certainly, in the both cases the results coincide.

The total process width Γ_p is obtained from the unitarity condition:

$$\Gamma_p = \frac{1}{T_0}(1 - |S_{ii}|^2) \approx \frac{1}{T_0}2ImT_{ii}, \quad (8)$$

$S = 1 + iT$. Here T_0 is the normalizing time, $T_0 \rightarrow \infty$. (In the general case $\Gamma_p = \Gamma_{\bar{n}} + \Gamma_a$, where $\Gamma_{\bar{n}}$ and Γ_a correspond to antineutron and annihilation mesons in the final state, respectively. For the $n\bar{n}$ conversion in nuclear matter $\Gamma_{\bar{n}} \approx 0$.)

In the impulse representation

$$\Gamma_p = -2Im\epsilon G\epsilon = 2Im\epsilon \frac{1}{V - i\Gamma/2}\epsilon. \quad (9)$$

Finally

$$\Gamma_p = \epsilon^2 \frac{\Gamma}{V^2 + (\Gamma/2)^2} \approx 4\epsilon^2/\Gamma. \quad (10)$$

Since all the antineutrons annihilate, we can compare Γ_p and Γ_f :

$$\frac{\Gamma_f}{\Gamma_p} = 1 + \left(\frac{\Gamma/2}{V}\right)^2 > 1. \quad (11)$$

The field approach gives reinforcement, in comparison with the phenomenological model. Moreover, the Γ -dependence of Eqs.(10) and (6) is inverse:

$$\frac{d\Gamma_f}{d\Gamma} > 0. \quad (12)$$

We stress that for any model of \bar{n} -medium interaction field approach gives $\Gamma_f \sim \Gamma$.

If $(\Gamma/2)^2 > V^2$ (the realistic set of parameters [4] fits this requirement),

$$\frac{d\Gamma_p}{d\Gamma} < 0. \quad (13)$$

At the point $(V = 0, \Gamma)$ Eq.(13) is true as well. Therefore, the phenomenological model gives the inverse Γ -dependence. At the same time annihilation is the main effect determining the process speed (see Eqs.(6) and (10)).

The phenomenological model is based on Eqs.(7) and (8). If instead of Hamiltonian (7) we take

$$\mathcal{H}_I = \mathcal{H}_{r,d} + \mathcal{H}, \quad (14)$$

$$\mathcal{H} = -i\Gamma/2\bar{\Psi}_{\bar{n}}\Psi_{\bar{n}}, \quad (15)$$

where \mathcal{H}_r and \mathcal{H}_d are the Hamiltonians of reaction and decay with \bar{n} in the final state, respectively, the qualitative conclusions do not change because the heart of the problem is in the term \mathcal{H} . In the next paper these problems will be studied in detail.

From Eq.(10) it is seen that real potential V leads to suppression, which is certainly correct. However, Γ acts in the same direction, which seems wrong at least at the threshold point $\Gamma = 0$ because the procedure $\Gamma = 0 \rightarrow \Gamma \neq 0$ implies the opening of a new channel of \bar{n} -medium interaction (annihilation). This must tend to increase Γ_p . Eq.(13) provides inverse tendency.

The reason for disagreement indicated above is the incorrect description of absorption on the whole, in particular the nonhermicity of Hamiltonian (7). Eq.(8) follows from the unitarity condition. Since the operator $iImU_{opt}$ is "antihermitian", we have

$$(SS^+)_{fi} = \delta_{fi} + \alpha_{fi}, \quad (16)$$

$\alpha_{fi} \neq 0$. Then instead of equation

$$\sum_{f \neq i} |T_{fi}|^2 \approx 2ImT_{ii} \quad (17)$$

one obtains

$$\sum_{f \neq i} |T_{fi}|^2 = 2ImT_{ii} - |T_{ii}|^2 + \alpha_{ii} \approx 2ImT_{ii} + \alpha_{ii}, \quad (18)$$

$\alpha_{ii} \neq 0$. Let us consider the right-hand side of this equation.

To get the expression for observable values we pass on to the evolution operator $U(t) = 1 + iT(t)$. Then

$$\sum_{f \neq i} |T_{fi}(t)|^2 \approx 2ImT_{ii}(t) + \alpha_{ii} = W(t), \quad (19)$$

where $W(t)$ is the process probability in a time t . α_{ii} can be neglected only when $|\alpha_{ii}| \ll 2ImT_{ii}$. However, for the value of $2ImT_{ii}(t)$ we have obtained (see Eqs.(8)-(10))

$$W(t) = \Gamma_p t = \frac{4\epsilon^2}{\Gamma} t. \quad (20)$$

We consider the $n\bar{n}$ transition in nuclear matter. We take $\Gamma = 100$ MeV, $t = T_0 = 1.3$ yr [6] (T_0 is the observation time in proton-decay type experiment) and $\epsilon = 1/\tau < 1/(10^8 s)$ [7,8]. As a result $2ImT_{ii} < 10^{-31}$. We believe that $10^{-31} \ll |\alpha_{ii}|$ (in our opinion $\alpha \sim 1$) and the basic equation $W(t) = 2ImT_{ii}(t)$ and thus Eq.(8) are meaningless.

Eq.(16) implies that: (a) Any matrix element S_{nm} (on-diagonal and off-diagonal) can contain some error. (b) The basic relation (8) is broken down. So the physical meaning of imaginary part of self-energy $Im\Sigma = -\Gamma/2$ (see Eq.(9)) is uncertain because it is clarified using the relation (8).

Strictly speaking, Eq.(8) can be used only for unitary S -matrix, or equations of Schrodinger type (when the interaction Hamiltonian contains the single term $\mathcal{H}_I = \mathcal{H}$) as in this case the problem is unitarized: by means of equation of motion and condition of probability conservation $1 = |U_{ii}(t)|^2 + W(t)$ the optical potential is fitted to experimental data. Certainly, the unitarity is a necessary and not a sufficient condition.

The particle absorption in the medium can be considered as a decay of one-particle state. With substitution

$$iImU_{opt} = -i\Gamma_x/2, \quad (21)$$

where Γ_x is a width of some free-space decay $\bar{n} \rightarrow x$, the formulas given above describe the decay in a final state (instead of absorption). Formally, in this case all the results are true as well.

Also we would like to stress the following. The importance of unitarity condition is well known [9,10]. Nevertheless, the nonhermitian models (7),(8) and (14),(8) are frequently used since they strongly simplify the calculations. In particular, all existing calculations of process (1) are based on model (7),(8) (see [5] for future references), resulting in the inverse Γ -dependence.

In the general case the total process probability corresponding to Eq.(14) is

$$W_t = W_{\bar{n}} + W_a, \quad (22)$$

where $W_{\bar{n}}$ and W_a are the probabilities to find antineutron and annihilation products, respectively. The phenomenological Hamiltonians like (14) containing several terms describe only $W_{\bar{n}}$. This result as well as other arguments in favour of tendency $d\Gamma_f/d\Gamma > 0$ will be presented in the next paper. The range of applicability of the complicated models containing the Hamiltonian \mathcal{H} will be studied as well.

The main result is as follows: the absorption (decay) in a final state does not tend to the process suppression. This is true for the reactions, decays and $n\bar{n}$ conversion in the medium. The field approach gives $\Gamma_f \sim \Gamma$, whereas models (7) and (8) give $\Gamma_p \sim 1/\Gamma$. Therefore, the calculations based on field approach (unitary models) can give a reinforcement of the corresponding processes (see Eq.(11)), in comparison with phenomenological model results.

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